Pure Core 3 Past Paper Questions Pack A: Mark Scheme

Taken from MAP1

January 2001

2 (a)	Sketch	B1	1	condone no scales for x and/or y
(b)	One root	B1	1	
(c)(i)	$f(4) \approx -0.61$, $f(5) \approx -0.61$	B1		numerical evidence needed; allow clear comparison of values of $6 - x$ and $\ln x$
	Change of sign $\Rightarrow 4 < \alpha < 5$	E1	2	AG; change of sign may not be mentioned but conclusion must be drawn
(ii)	$f(4.5) \approx -0.004$	B1		evidence needed; allow comparison
	Negative value $\Rightarrow \alpha$ nearer to 4	E1F	2	f.t positive value for f(4.5)
	Total		**6**	

Q	Solution	Marks	Total	Comments
4 (a)	Translation two units in positive x direction	B1		
	Translation two units in negative y direction	B1	2	
(b)	Sketch of $f - g$	B2,1,0	2	B1 if one error made – e.g. whole graph translated downwards condone no scales or wrong scales for x and y condone part of graph invisible but clearly intended to be $y = 0$
(c)(i)	f has no inverse function	B1		
(ii)	g is even	B1		
(iii)	Range of h is $0 \le y \le 4$	B1F	3	f.t from c's sketch which scored B1
(d)(i)	$f(x) < 2 \Leftrightarrow 0 < x < 4$	B1		allow answers without working shown here
(ii)	$g(x) < 2 \Leftrightarrow -4 < x < 4$	B1		use of $\le$ for $<$ penalty 1 mark
(iii)	$f(x) > g(x) \Leftrightarrow x < 2$	B2	4	
	Total		**11**	

June 2001

7	**a**	**Either**		
		$x > 4$ mentioned as part of solution	B1	Allow even if this is c's full solution but $\lvert x \rvert > 4$ earns 0 out of 3
		$x = 2$ mentioned as a critical value	B1	or sketch of $y = \lvert x - 3 \rvert$ and $y = 1$
		Soln set $x < 2$ or $x > 4$	B1F 3	not $\le$, $\ge$; not $4 < x < 2$; ft wrong value
		Or		
		Critical values 2 and 4	M1A1	from sketch or equations
		Soln set as above	A1F (3)	ft wrong critical values

8	a	Reflection in y-axis	B1		OE
		Stretch with scale factor 2 ...	B1		
		... parallel to y-axis	B1	3	
	b	Attempt to reflect in $y = x$	M1		Accept c's intention of doing so
		Sketch showing inverse function	A1	2	with pos y-axis as asymp and crossing x-axis
	c	Use of ln as inverse of exp	M1		perhaps seen on arrow diagram
		$y = 2e^{-x} \Rightarrow \ln y = \ln 2 + \ln e^{-x}$	m1		OE, eg $\ln\frac{y}{2} = -x$
		$f^{-1}(x) = \ln 2 - \ln x$	A1	3	OE, eg $-\ln\frac{x}{2}$ - must be in terms of x
	d	Domain of f^{-1} is $x > 0$	B1		
		Range of f^{-1} is all real numbers	B1	2	
	e	$f(t) = 0.5$	M1		PI
		Use of ln as inverse of exp	m1		or $t = f^{-1}(0.5)$ using answer to (c), but logarithms must be used
		ie 1.39 hours after the injection	A1	3	Allow 2 ln 2 or AWRT 1.4; condone omission of units; NMS 2/3

January 2002

Q	Solution	Marks	Total	Comments
4 (a)(i)	Stretch parallel to x-axis	B1		accept 'squashed' with SF 2
	SF$\frac{1}{2}$	B1	2	1/2 for right idea without using the language of geometrical transformations
(ii)	L is $x = \frac{\pi}{2}$	B1	1	condone $\frac{\pi}{2}$, $y = \frac{\pi}{2}$ etc, and/or degrees
(b)(i)	Range is $-1 \le f(x) \le 1$	B1	1	allow any symbol for $f(x)$ but not $<$ for $\le$
(ii)	Domain is $-1 \le x \le 1$	B1F		ft wrong answer to part (i)
	Range is $0 \le f^{-1}(x) \le \frac{\pi}{2}$	B1	2	condone 90 for $\frac{\pi}{2}$
(iii)	Attempt to reflect in $y = x$	M1		must be a curve
	Correct sketch	A1	2	ignore anything shown outside range; ignore scales
(c)(i)	$gf(x) = \lvert\cos 2x\rvert$	B1	1	
(ii)	Suitable reflection of part of graph	M1		condone smooth curve without cusp
	All correct with cusp	A1	2	ignore anything shown outside domain; ignore scales
	Total		**11**	

Q	Solution	Marks	Total	Comments
6 (a)(i)	$e^x = 0$ impossible	E1		PI by working in part (ii)
	$2e^x - 3 = 0$ is the only possibility	E1	2	
(ii)	$x = \ln\frac{3}{2}$ at SP	B1		accept AWRT 0.40 or 0.41 even if NMS
	Attempt to find y	M1		using $e^x = \frac{3}{2}$
	$y = -\frac{1}{4}$	A1	3	NMS Award B1 for AWRT –0.25
(iii)	Deriv of $2e^{2x}$ is ke^{2x}	M1		M0 B0 if c differentiates each factor and multiplies
	Deriv of $-3e^x$ is $-3e^x$	B1		
	$y'' = \frac{9}{2}$ at SP	A1		accept AWRT 4.5
	so it is a min point	A1F	4	ft wrong value of y'' at c's x (must have value of x)
(b)(i)	$e^x = 1$ or 2	B1		allow verification here (B1 for each value)
	$\Rightarrow x = 0$ or $\ln 2$	B1	2	convincingly found (AG)
(ii)	$\int y\,dx = ke^{2x}(+c)$	M1		M0 if c integrates each factor and multiplies
	$\int y\,dx = \frac{1}{2}e^{2x} - 3e^x + 2x(+c)$	A1		
	$\int_0^{\ln 2} y\,dx = (2 - 6 + 2\ln 2) - (\frac{1}{2} - 3 + 0)$	m1		attempt to substitute and subtract (all terms)
	$... = -\frac{3}{2} + 2\ln 2$	A1		
	Area below axis, hence result (AG)	E1	5	allow correct use of minus or mod signs early in working but not a last-minute unexplained change of sign
(c)	Second symbol cannot be replaced	B1		allow B1 even if unexplained
	Reason (constant of integration)	E1	2	allow E1 even if not entirely precise
	Total		**18**	

June 2002

3 (a)	$\frac{d}{dx}\left(2x^{\frac{1}{2}}\right) = kx^{-\frac{1}{2}}$ or $\frac{d}{dx}(\ln x) = \frac{1}{x}$	M1		
(i)	$k = 1$	A1		Allow $2 \times \frac{1}{2}$
(ii)	$\frac{d}{dx}(\ln(x+1)) = \frac{1}{x+1}$	A1	3	
(b)	$\int\left(x^{-\frac{1}{2}} + \frac{1}{x+1}\right)dx = 2x^{\frac{1}{2}} + \ln(x+1)$	M1		Allow M1 if at least one term correct
	Substituting $x = 4$ or $x = 1$	m1		in at least one correct term
	Both substitutions and subtraction	m1		ditto; condone subtraction wrong way round.
	Use of log law	m1		Accept $(4 + \ln 5) - (2 + \ln 2) = 2 + \ln\frac{5}{2}$
	Answer $2 + \ln\frac{5}{2}$	A1	5	convincingly found (AG)
	Total		**8**	

Q	Solution	Marks	Total	Comments
4 (a)	$y' = pe^{-2x}$	M1		Where p is a constant, $p = \pm 2$ or $\pm\frac{1}{2}$ or ± 1
	$p = -2$	A1		
	$y'' = 4e^{-2x}$	A1F	3	ft consistent errors provided $p \neq 1$
(b)(i)	$\int y\,dx = qe^{-2x}(+c)$	M1		Where q is a constant, $q = \pm 2$ or $\pm\frac{1}{2}$ or ± 1
	$q = -\frac{1}{2}$	A1F	2	ft wrong value of p provided $p \neq 1$
(ii)	Area $= \int_0^1 y\,dx$	M1		Allow even if formula not used; condone $\int_0^0 y\,dx$
	$\ldots = -\frac{1}{2}e^{-2} + \frac{1}{2}$	A1F		$e^0 = 1$ must be used; ft wrong coefficient in (i)
	$\ldots = \frac{e^2 - 1}{2e^2}$	A1	3	convincingly obtained (AG)
	Total		**8**	

7(a)(i)	Reflection in $y = x$	B1	1	
(ii)	Good attempt at reflection in $y = x$	M1		ie correct shape between intersections
	Correct intersections with $y = x$	A1	2	
(b)(i)	Stretch parallel to y-axis	B1		
	SF 3	B1	2	
(ii)	e^z seen	M1		Where z is any function of x or y
	$y = 3\ln x \Rightarrow x = e^{\frac{1}{3}y}$	A1		OE
	$f^{-1}(x) = e^{\frac{1}{3}x}$	A1	3	NMS 3/3
	Total		**8**	

November 2002

Q	Solution	Marks	Total	Comments
1 (a)	Equating and clearing denominator	M1		At least 2 of 3 terms multiplied by x
	All correct	A1	2	AG
(b)	$f(0.6) \approx -0.184$, $f(0.7) \approx 0.043$	B1B1		OE using original functions
	Sign change, so root between	E1	3	Sign change OE must be mentioned
	Total		**5**	

3 (a)	$\int x^{\frac{3}{2}}\,dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}}(+c)$	M1A1		M1 for attempt at $\frac{x^{n+1}}{n+1}$
	Substitution and subtraction	m1		Subtraction must be the right way round
	$\int_1^4 x^{\frac{3}{2}}\,dx = \frac{4^{\frac{5}{2}}}{\frac{5}{2}} - \frac{1^{\frac{5}{2}}}{\frac{5}{2}} = \frac{62}{5}$	A1	4	AG but allow evaluation on calculator
(b)	Use of $\int \frac{1}{x}\,dx = \ln x\,(+c)$	M1		Condone misuse of the 2 here
	$\int_2^{18} \frac{1}{2x}\,dx = \frac{1}{2}(\ln 18 - \ln 2)$	A1		OE
	Use of $\ln a - \ln b = \ln \frac{a}{b}$	m1		
	$\int_2^{18} \frac{1}{2x}\,dx = \frac{1}{2}\ln 9 = \ln 3$	A1	4	
	Total		**8**	

7 (a)	Reflection in y-axis	B1		
	Stretch in y direction	B1		
	...SF 2	B1	3	
(b)(i)	Range of f is $0 < f(x) \le 2$	B1B1	2	Allow any symbol for f(x); 1/2 for 'from 0 to 2' OE
(ii)	Domain of f^{-1} is $0 < x \le 2$	B1F		Allow any symbol; ft wrong answer to (i)
	Range is $f^{-1}(x) \ge 0$	B1	2	Allow any symbol
(iii)	$\ln z$ appearing in c's solution	M1		where z is any function of x or y
	Use of division by 2	m1		intended as inverse of multiplication by 2
	$f^{-1}(x) = -\ln\frac{x}{2}$	A1	3	OE, eg $\ln 2 - \ln x$ or $\ln\frac{2}{x}$
(iv)	f(ln 2)=1	B1		Stated or implied
	Implication is correct (reason)	E1	2	Dependent on B1 Condone imperfect reasoning
	Total		**12**	

January 2003

Q	Solution	Marks	Total	Comments
5 (a)(i)	Coordinates are (0, 5)	B1	1	Condone $y = 5$
(ii)	Range of f is $f(x) > 0$	B1	1	Allow any clear notation for $f(x)$ B0 for $y \geq 0$ or 'from 0 to ∞'
(iii)	$f(\ln 6) = \frac{5}{6}$	B2, 1	2	Allow NMS; condone use of decimals Allow 1/2 for AWRT 0.833
(b)(i)	$gf(x) = 5e^{-x} + 10 = 5(e^{-x} + 2)$	B1	1	Convincingly shown (AG)
(ii)	Range of gf is $gf(x) > 10$	B1	1	Allow any clear notation for $gf(x)$ Condone $y \geq 10$ or 'from 10 to ∞'
(iii)	Decreasing exponential-type curve	M1		
	y-intercept 15 or asymptote $y = 10$	A1	2	
(iv)	$gf(x) = 11 \Rightarrow 5e^{-x} = 1$	B1		
	Attempt to take logs	M1		
	$\ldots \Rightarrow x = \ln 5$	A1	3	Convincingly obtained (AG) SC Reverse reasoning: max 1/3
(c)(i)	Initial temp 15 °C	B1	1	Condone absence of units
(ii)	$5(e^{-t} + 2) = 11$ OE stated	M1		
	Time is $\ln 5 \approx 1.6$ min	A1	2	Condone absence of units; accept AWRT 1.6; allow NMS
	Total		**14**	

Q	Solution	Marks	Total	Comments
6 (a)(i)	$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$	M1A1	2	M1 if coefficient or index correct
(ii)	Gradient at $x = 4$ is $\frac{1}{4}$	A1F	1	ft wrong coeff
(b)(i)	$\int f(x)dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}}...$	M1A1		M1 for $kx^{\frac{3}{2}}$
	$...+ 2x\ (+c)$	B1	3	
(ii)	Substituting $x = 4$	M1		In c's integral (not f(x) or f'(x))
	$\int_0^4 f(x)dx = \frac{40}{3}$	A1	2	Convincingly found (AG)
(c)	$y = x^{\frac{1}{2}} + 2 \Rightarrow x^{\frac{1}{2}} = y - 2$	M1		OE
	$... \Rightarrow x = (y-2)^2$, hence result	A1	2	Convincingly shown (AG)
(d)(i)	Line of symmetry is $y = x$	B1	1	
(ii)	Complete method for area of A	M2, 1		M1 for area of some relevant region (not just a rectangle or triangle) or $\int_2^4 (x-2)^2\ dx$
	Shaded area is $\frac{32}{3}$	A2,1	4	A1 for area of relevant region or $... = \frac{8}{3}$ or if c makes one error after M2 SC M1A1 for $\int_0^4 f(x)dx - \int_0^4 f^{-1}(x)dx = 8$
	Total		**15**	

June 2003

4 (a)	$\sin^2 x + \cos^2 x \equiv 1$ stated	M1		or used
	$2\sin^2 x + \sin x = 0$	A1	2	convincingly shown (AG)
(b)	$\sin x = 0$ or $-\frac{1}{2}$	B1B1		
	$\sin x = 0 \Rightarrow x = 0$ or π	B1		In (b) condone degrees or decimals, and ignore values outside domain B0 if other values in domain included
	Use of $\sin\frac{\pi}{6} = \frac{1}{2}$ OE	M1		PI
	$\sin x = -\frac{1}{2} \Rightarrow x = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$	A1A1	6	Deduct 1 for each incorrect value given (in domain) NMS 4/4
	Total		**8**	

Q	Solution	Marks	Total	Comments
5(a)(i)	$y' = \frac{1}{x}$	B1	1	
(ii)	When $x = \mathrm{e}$, $y' = \frac{1}{\mathrm{e}}$	B1	1	NMS 1/1 for AWRT 0.368 or 0.367 after $y' = \frac{1}{x}$
(b)	Translation (in x or y direction)	M1		Allow 'transformation' if clarified
	2 units in positive y direction	A1	2	Condone 'by a factor of 2'
(c)(i)	Range of f is all real numbers	B1	1	
(ii)	Domain of f^{-1} is all real numbers	B1F		ft wrong answer to (c)(i)
	Range is $\mathrm{f}^{-1}(x) > 0$	B1	2	Condone $\mathrm{f}^{-1}(x) \geq 0$ or 'from 0 to ∞' Allow any symbol for $\mathrm{f}^{-1}(x)$ but it must be clear which is which
(iii)	e^z appearing in solution	M1		where z is any function of x or y
	Use of -2 as inverse of $+2$	m1		PI
	$\mathrm{f}^{-1}(x) = \mathrm{e}^{x-2}$	A1	3	
(d)(i)	$\mathrm{fg}(x) = 2 + \ln(\mathrm{e}x^3)$	M1		
	$\ln(\mathrm{e}x^3) = \ln \mathrm{e} + \ln x^3$	m1		OE
	$\mathrm{fg}(x) = 3(1 + \ln x)$	A1	3	convincingly shown (AG)
(ii)	$\mathrm{fg}(x) = 9 \Rightarrow \ln x = 2$	M1		M1A0 for verification
	$\ldots \Rightarrow x = \mathrm{e}^2$	A1	2	AG
	Total		**15**	

November 2003

Q	Solution	Marks	Total	Comments
7 (a)	Right shape for sketch	M1		With vertex on x-axis, $x < 0$
	Right relationship to given graph	A1	2	Given graph must be copied for this mark
(b)	Yes, sufficient (reason)	E2,1	2	E1 if reason imperfectly expressed
(c)	Solution is $x < 0$	B1	1	
				SC If $y = \lvert 2x - 1 \rvert$ sketched, max B1 E1 ('No' with reason) B1 $(x > 0)$
	Total		**5**	

January 2004

Q	Solution	Marks	Total	Comments
5 (a)	$y' = 2e^{2x} \ldots$	M1A1		M1 for ke^{2x}
	$\ldots - 2x^{-2}$	B1	3	
(b)	At SP $2e^{2x} = 2x^{-2}$	m1		OE
	Multiplication by x^2	m1		Dep on m1
	$x^2 e^{2x} = 1$	A1	3	convincingly shown (AG)
(c)	Take square roots, $xe^x = 1$	B1		AG (square roots must be mentioned); condone no mention of ±
	Then take logs, $\ln x + x = 0$	M1A1	3	AG; M1 for use of a log law or $\ln e^x = x$ or $\ln 1 = 0$
(d)	$f(0.5) \approx -0.19, f(0.6) \approx 0.09$	B1B1		Where $f(x) = \ln x + x$
	Change of sign, so root between	E1	3	AG
(e)	$\int \left(e^{2x} + 2x^{-1}\right) dx = \frac{1}{2}e^{2x}$	M1A1		M1 for ke^{2x}
	$\ldots + 2 \ln x \ (+c)$	B1	3	Modulus not needed here
		Total	**15**	

6(a)(i)	$fg(x) = \sqrt{x-1}$	B1		
	$gf(x) = \sqrt{x-1}$	B1	2	
(ii)	$fg(1) = gf(1) = 0$	B1	1	
(b)(i)	Translation	M1		Accept 'transformation' if clarified
	1 unit in (positive) x direction	A1	2	'Positive' may be implied
(ii)	Range of h is $0 \leq h(x) \leq 2$	B1	1	Allow any symbol for $h(x)$; condone < for ≤; allow '0 to 2'
(iii)	Domain of h^{-1} is $0 \leq x \leq 2$	B1F		ft wrong answer in (ii); any symbol for x
	Range of h^{-1} is $1 \leq h^{-1}(x) \leq 5$	B1	2	Allow any symbol for $h^{-1}(x)$; condone < for ≤; allow '1 to 5'
(iv)	$y = \sqrt{x-1} \Rightarrow y^2 = x - 1$	M1		OE
	$\ldots \Rightarrow x = y^2 + 1$	m1		Condone sign error here
	So $h^{-1}(x) = x^2 + 1$	A1	3	Allow NMS 3/3
		Total	**11**	

June 2004

3(a)	$y(0) = 6, \ y(1) = -1$	B1B1		
	Sign change, so root between	E1	3	

Q	Solution	Marks	Total	Comments
7(a)(i)	$\int\left(e^{2x}+1\right)dx=\frac{1}{2}e^{2x}+x(+c)$	M1A1 A1	3	M1 for at least one term correct
(ii)	Substitution and subtraction	M1		In c's integral (not in y or y') Subtraction the right way round
	$\int_0^{\ln 2}\left(e^{2x+1}\right)dx=(2+\ln 2)-\frac{1}{2}$	A1		Allow if the first term (2) is correct
	$=\frac{3}{2}+\ln 2$	A1	3	Convincingly shown (AG)
(b)(i)	$x=0\Rightarrow y=2$	B1	1	
(ii)	Use of $e^{\ln 2}=2$ or $e^{\ln 4}=4$	M1		
	$x=\ln 2\Rightarrow y=5$	A1	2	NMS 2/2 for AWRT 5.00
(c)(i)	Range of f is $2\le f(x)\le 5$	B1F	1	ft wrong answers in (b); condone < for ≤; allow any notation for f(x)
(ii)	Sketch of f with correct domain	B1		
	Sketch of inverse fn correct	B1	2	Ignore anything outside domain; curve must intersect positive x-axis
(iii)	ln z appearing in solution	M1		Where z is any function of x or y
	Complete method	m1		
	$f^{-1}(x)=\frac{1}{2}\ln(x-1)$	A1	3	correctly bracketed and in terms of x; NMS 3/3
	Total		**15**	